Stabilization of the Two Wheels Inverted Pendulum by a Nested Saturation Function

O. Octavio Gutiérrez-Frías[1] and A. Luviano-Juarez[2] and E. Rodriguez-Hernandez[2]
[1] CECyT No. 1 "GVV", Instituto Politcnico Nacional
GAM 07480, México D.F.
ogutierrezf@ipn.mx
Phone: (52)-55-57296000
[2] UPIITA, Instituto Politcnico Nacional
GAM 07340, México D.F.
aluvianoj@ipn.mx, erodriguezh0700@ipn.mx
Phone: (52)-55-57296000

***Abstract*— The stabilization of the Two Wheels Inverted Pendulum around its unstable equilibrium point is presented in this paper. The control strategy describes the underactuated system as a chain of integrator with a nonlinear perturbation, allows us to use a nested saturation control technique for making all state variable converges to zero. The proposed controller makes the closed loop system asymptotically stable and locally exponentially stable around its unstable top position, under the assumption that the pendulum is initially above the horizontal plane.**

Keywords: Nonlinear Control, Underactuated System, Nested Saturation function, Lyapunov Method.

I. Introduction

The stabilization of the Two Wheels Inverted Pendulum (**TWIP**) is an interesting problem in nonlinear control, as seen by growing list of articles, for examples: (Baloh et.al., 2003), (Salerno and Angeles, 2003), (Kim et. al, 2005), (Karla et. al., 2007), (Ren et. al, 2008),(Viguria et. al., 2006), (Vermeiren et. al., 2011), to mention only a few references. The device consists of a free vertical rotating pendulum with two wheels actuated by DC motors, where the torque of the motors is used as input control. The main control objective is to drive the pendulum to the rest upright position with the wheels to the rest origin. Since the angular acceleration of vertical pendulum cannot be directly controlled, the **TWIP** is considered an underactuated mechanical system. Besides, this nonlinear system allows us to illustrate a simplified version of the well known *Segway* (Segway, 2011).

There are many works related with the problem control of this system, for example: Grasser et. al. (Grasser et. al., 2002) designed a prototype of a revolutionary two-wheeled vehicle called ”JOE”. The control system is based on two state-space controllers. In (Jeong and Takahashi, 2008) the authors proposed the design concept of the human assistant robot I-PENTAR (Inverted PENdulum Type Assistant Robot) aiming at the coexistence of safety and work capability and its mobile control strategy. In (Shimada and Hatakeyama, 2008), the authors introduced a high speed robust motion control technique for the system based on the concept of instability. Khac Duc Do and Gerald Seet (Do and Seet et. al., 2010) presented a nested saturation control design technique that it is applied to derive a control law for a two-wheeled vehicle with an inverted pendulum. Exist other works related with the mobile inverse pendulum, we recommend the following references: (Pathak et. al., 2005),(Nawawi et. al., 2008),(Noh et. al., 2010), (Huang et. al., 2010) and (Strah et. al., 2010).

In this paper, we propose a strategy to stabilize for the Two Wheels Inverted Pendulum around its the unstable top position. Inspired by the procedure presented in (Aguilar et. al., 2011), we transform the original system into nonlinearly perturbed chain of four chain and then, we introduced a controller based on nested saturation functions. Next, we show that the closed loop solution is bounded, which allow to prove that system is locally exponentially estable. The stability analysis is fairly simple because it is carried out using the Lyapunov method.

The paper is organized as follows. In Section II, we present the model of the **TWIP** and the transformation of the original system into an integrators chain. Section III is devoted to obtain a nonlinear controller for the stabilization of the **TWIP**. The analysis stability of the closed-loop system is also presented in the same section. In Section IV, we show some computer simulation. Finally, we devote Section V to conclusions.

II. The Two Wheels Inverted Pendulum

The **TWIP**, depicted in the Figure 2, is a free rotating vertical pendulum with two wheels actuated by DC motors. The model of this system is described by (Gutirrez, 2011).

$$\begin{aligned}&(\delta+1)\mu\ddot{\theta}+\cos\gamma\ddot{\gamma}-\dot{\gamma}^2\sin\gamma=u\\&\mu\cos\gamma\ddot{\theta}+\eta\ddot{\gamma}-\sin\gamma=0\end{aligned}\tag{1}$$

where θ is rotational angle of wheels; γ is the angle of the pendulum; u is the normalized voltage applied

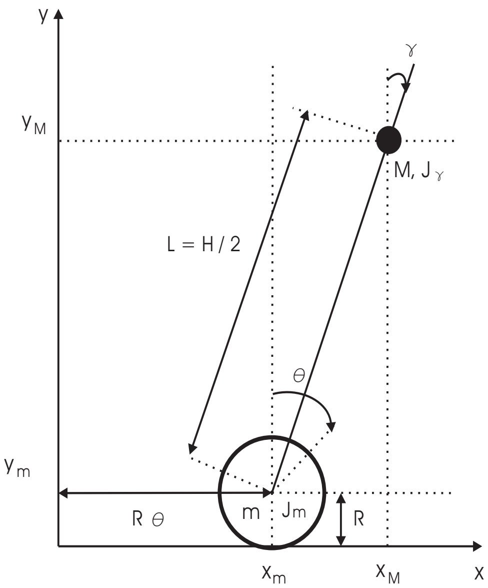

Figure 1. Two Wheels Inverted Pendulum System

to the wheels (i.e. input control), δ is a constant that depends directly on both, wheels and pendulum mass, μ is related with the radius of the wheels and the length of the pendulum and the parameter η depends on mass and inertia of the pendulum. Also, the ”.”stand for differentiation with respect dimensionless time.

To the simplify the algebraic manipulations in the forthcoming development, we derive the control variable τ.

$$u = \left((\delta+1)\mu - \frac{\mu}{\eta}\cos^2\gamma\right)\tau + \frac{1}{\eta}\cos\gamma\sin\gamma - \dot{\gamma}^2\sin\gamma \tag{2}$$

Therefore, the normalized system (1) is equivalent to the following feedback system:

$$\begin{aligned} \ddot{\gamma} &= \tfrac{1}{\eta}\sin\gamma - \tfrac{\mu}{\eta}\cos\gamma\tau \\ \ddot{\theta} &= \tau \end{aligned} \tag{3}$$

if $\tau = 0$ and $\gamma \in [0, 2\pi]$, then, the system has two equilibrium points: one is an unstable equilibrium point $q = (0, 0, 0, 0)$ and the other is a stable equilibrium point $q = (\pi, 0, 0, 0)$.

Now, in order to express the system (3) as a chain of integrator plus an additional perturbation. From (Aguilar et. al., 2011), we proposed the following coordinates:

$$\begin{aligned} x_1 &= \theta + 2\tfrac{\eta}{\mu}\arctan h\left(\tan(\tfrac{\gamma}{2})\right) \qquad & x_3 &= \tan\gamma \\ x_2 &= \dot{\theta} + \tfrac{\eta}{\mu}\dot{\gamma}\sec\gamma & x_4 &= \dot{\gamma}\sec^2\gamma \end{aligned} \tag{4}$$

Then, the system (3) can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \tfrac{1}{\mu}x_3 + \tfrac{\eta}{\mu}\tfrac{x_3}{(1+x_3^2)^{3/2}}x_4^2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= v_f \end{aligned} \tag{5}$$

and the new controller v_f is defined as

$$v_f = \sec^2\gamma\left[\frac{1}{\eta}\sin\gamma - \frac{\mu}{\eta}\cos\gamma\tau + 2\dot{\gamma}^2\tan\gamma\right] \tag{6}$$

Clearly, the last set of differential equation can be expressed as

$$\dot{x} = Ax + Bv_f + \Psi(x3, x4) \tag{7}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\mu} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; \tag{8}$$

and

$$\Psi(x3, x4) = \frac{\eta}{\mu}\frac{x_3}{(1+x_3^2)^{3/2}}x_4^2 \tag{9}$$

Comment 1 The above system has a similar form to the four cascade integrator with additional nonlinear perturbation, this representation has been used in (Lozano and Dimogianolopus, 2003; Aguilar and Gutirrez, 2008). On the other hand, the control v_f and the non-actuated coordinate γ are not completely uncoupled, this means that the control act directly on the additional nonlinear perturbation.

Problem Statement: The control objective is to design a controller τ to bring the pendulum to the upright position with the wheels position at the origin assuming that the pendulum is initially above the horizontal plane.

III. CONTROL STRATEGY

In this section, we establish the framework of our control strategy. The idea consists of bringing all the state very close to the origin, for this purpose we use a nested saturation based controller. This technique, introduced in (Teel, 1992) by Teel, has been used for controlling a wide class of the under actuated system (Teel, 1993; Castillo et. al., 2005; Aguilar and Gutirrez, 2008; Aguilar et. al., 2009).

Thus, our stability problem will be solved as follows. First, a linear transformation is used to directly propose a stabilizing controller; then, it is show that the proposed controller guarantees the boundedness of all states. Finally, we show that the closed loop system is locally exponentially asymptotically stable after some finite time.

III-A. A nested based Controller

Then, we introduce some convenient definition that we are using in the development of our control strategy.

Definition 1: Let $p \in \Re$. The linear saturation function is defined as

$$\sigma_r(p) = \begin{cases} p & if\ |p| \leq r \\ rsign(p) & if\ |p| > r \end{cases} \tag{10}$$

Inspired on the work presented in (Teel, 1992), we propose a convenient transformation $q = Sx$ [1]that allows us to obtain, in a direct way, the stabilizing controller v_f for the nonlinear system (5), such that:

$$SAS^{-1} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad SB = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{11}$$

Then, we can propose S as:

$$S = \begin{bmatrix} \mu & 3\mu & 3 & 1 \\ 0 & \mu & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

So, system (5) is transformed as:

$$\begin{array}{l} \dot{q}_1 = q_2 + q_3 + q_4 + 3\eta\phi(x_3)q_4^2 + v_f \\ \dot{q}_2 = q_3 + q_4 + \eta\phi(x_3)q_4^2 + v_f \\ \dot{q}_3 = q_4 + v_f \\ \dot{q}_4 = v_f \end{array} \tag{13}$$

where $\phi(x_3) = \frac{x_3}{(1+x_3^2)^{3/2}}$ [2]

To stabilize the above system, we propose the following nested based controller v_f as:

$$v_f = -q_4 - q_3 - \sigma_\beta\left(q_2 + \sigma_\alpha(q_1)\right) \tag{14}$$

III-B. Boundness of all states

We show in four steps that the closed loop system (13) and (14), ensures that all states are bounded, furthermore, the bound of each state depends directly on controller parameters described in (14).[3]

First step: To show that q_4 is bounded, we define an auxiliary positive function V_4 as:

$$V_4 = \frac{1}{2}q_4^2 + \frac{1}{2}(q_3 - q_4)^2 \tag{15}$$

Then, differentiating V_4 and using the system (13) and controller (14), we have:

$$\dot{V}_4 = -2q_4^2 - q_4\sigma_\beta\left(q_2 + \sigma_\alpha(q_1)\right)$$

[1]we use $q = [q_1, q_2, q_3, q_4]^T$ and $x = [x_1, x_2, x_3, x_4]^T$

[2]It is easy verify that function $|\phi(x_3)| \leq 2/3^{3/2} = \overline{K}$.

[3]Note that the closed loop system (13) and (14) is locally Lipschitz. consequently $\{q_1, q_2 q_1, q_2, q_4\}$ cannot have a finite time scape (Khalil, 2002).

If $|q_4| \geq \beta/2 = \varepsilon$, then, we have that $\dot{V}_4 < 0$, therefore, there is a finite time $T_1 > 0$ such that

$$|q_4| < \varepsilon; \forall t > T_1$$

Second Step: Now, we proceed to analyze the behavior of the state q_3, Therefore, we introduce an auxiliary positive function V_3.

$$V_3 = \frac{1}{2}q_3^2 \tag{16}$$

Differentiating Equation (16), we obtain after substituting the proposed controller (14) into the third differential equation of system (13)

$$\dot{V}_3 = -q_3^2 - q_3\sigma_\beta\left(q_2 + \sigma_\alpha(q_1)\right) \tag{17}$$

If $|q_3| > \beta$, then, we have that $\dot{V}_3 < 0$. Therefore, there is a finite time $T_2 > T_1$, after which

$$|q_3| < \beta; \forall t > T_2$$

Third Step: Substituting Equation (14) into the second differential equation of system (13), we obtain

$$\dot{q}_2 = -\sigma_\beta\left(q_2 + \sigma_\alpha(q_1)\right) + \eta\phi(x_3)q_4^2 \tag{18}$$

In order show that q_2 is bounded, we propose the auxiliary positive function $V_2 = \frac{1}{2}q_2^2$.

Differentiating V_2 and using (18), it yields

$$\dot{V}_2 = -q_2\left(\sigma_\beta\left(q_2 + \sigma_\alpha(q_1)\right) - \eta\phi(x_3)q_4^2\right) \tag{19}$$

where β and α must satisfy $\beta > 2\alpha + \eta\overline{K}\varepsilon^2$ [4]

Evidently, If $|q_2| > \alpha + \eta\overline{K}\varepsilon^2$, then, we have that $\dot{V}_2 < 0$ and there is a finite time $T_3 > T_2$, after which

$$|q_2| < \alpha + \eta\overline{K}\varepsilon^2 \forall t > T_3$$

Consequently, q_2 is bounded and the control v_f turns out to be

$$v_f = -q_4 - q_3 - q_2 + \sigma_\alpha(q_1) \tag{20}$$

Fourth Step: Substituting Equation (20) into the first differential equation of system (13), we obtain

$$\dot{q}_1 = -\sigma_\alpha(q_1) + 3\eta\phi(x_3)q_4^2 \tag{21}$$

Now, we define a auxiliary positive function $V_1 = \frac{1}{2}q_1^2$.

Differentiating V_1 and using (21), we have

$$\dot{V}_1 = -q_1\left(\sigma_\alpha(q_1) - 3\eta\phi(x_3)q_4^2\right) \tag{22}$$

where α must satisfy $\alpha > 3\eta\overline{K}\varepsilon^2$. If $|q_1| > 3\eta\overline{K}\varepsilon^2$, then, $\dot{V}_1 < 0$ and Hence, there is a finite time $T_4 > T_3$, such that:

$$|q_1| < 3\eta\overline{K}\varepsilon^2 \forall t > T_4$$

[4]Notice that after $t > T_3$, it has $|\phi(x_3)|q_4^2 \leq \overline{K}\varepsilon^2$

Consequently, q_1 is bounded.
Thus, all the previous constraints on parameters β and α can be summarized as:

$$\beta > 2\alpha + \eta\overline{K}\varepsilon^2 \quad \alpha > 3\eta\overline{K}\varepsilon^2 \tag{23}$$

Hence, manipulating the last inequalities, we can selected the control parameters as follows:

$$\beta = \tfrac{4\lambda_1}{7\eta\overline{K}} \quad \alpha = 3\lambda_2\eta\overline{K}\tfrac{\beta^2}{4} \tag{24}$$

where$0 < \lambda_1 \leq 1$ and $\lambda_2 > 1$

III-C. Convergence to the origin of the whole states

We will prove that the closed loop system defined by (13) and (14) is asymptotically stable and locally exponentially stable, provided that the controller parameters satisfies the inequalities (23).

We must note that after $t > T_4$, the control law is no longer saturated, that is

$$v_f = -q_1 - q_2 - q_3 - q_4 \tag{25}$$

and the closed loop system can be expressed as:

$$\begin{aligned} \dot{q}_1 &= -q_1 + 3\eta\phi(x_3)q_4^2 \\ \dot{q}_2 &= -q_1 - q_2 + \eta\phi(x_3)q_4^2 \\ \dot{q}_3 &= -q_1 - q_2 - q_3 \\ \dot{q}_4 &= -q_1 - q_2 - q_3 - q_4 \end{aligned} \tag{26}$$

Let us define the following Lyapunov function

$$V = \frac{1}{2}q^T q \tag{27}$$

Now, differentiating V along the trajectories of (26), we obtain

$$\dot{V} = -qMq + (3\eta q_1 + \eta q_2)\phi(x_3)q_4^2 \tag{28}$$

where M is given by

$$M = \begin{bmatrix} 1 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1 \end{bmatrix}$$

Note that $\lambda_{min} = 1/2$ and therefore $M > 0$.

Then, we easily show that the second term of the right hand of Equation (28) satisfies

$$\begin{aligned} |(3\eta q_1 + \eta q_2)\phi(x_3)q_4^2| &< \frac{\overline{K}}{2}|(3\eta q_1 + \eta q_2)q_4^2|; \\ &< \frac{\overline{K}}{2}((3\eta q_1 + \eta q_2)^2 q_4^4|. \end{aligned} \tag{29}$$

So, applying the inequality (29) into time derivative of V (28), we have

$$\dot{V} < -\frac{1}{2}(q_1^2 + q_2^2 - \overline{K}(3\eta q_1 + \eta q_2)) - \frac{1}{2}q_3^2 - \frac{1}{2}q_4^2(1 - \overline{K}q_4^2) \tag{30}$$

Hence, we obtain that the previous inequality is strictly negative definite, since

$$q_1^2 + q_2^2 - \overline{K}(3\eta q_1 + \eta q_2) > 0 \tag{31}$$

and

$$1 - \overline{K}q_4^2 \geq 1 - \overline{K}\frac{\beta^2}{4} > 0 \tag{32}$$

Therefore, $\dot{V}$ is strictly negative definite and the vector state q locally exponentially converges to zero after $t > T_4$.

From the above discussion, we have

Proposition 1: Consider the closed loop system of the TWIP as described by model (5) with:

$$v_f = -2x_4 - x_3 - \sigma_\beta(\mu x_2 + 2x_3 + x_4 + \sigma_\alpha(\mu x_1 + 3\mu x_2 + 3x_3 + x_4)) \tag{33}$$

Then, the closed loop system is asymptotically stable and locally exponentially stable provided that the control parameters β and α satisfy the inequalities (23).

IV. Simulations Results

In order to test the performance of the proposed nonlinear control strategy, we developed an experiment that allow us to compare the behavior of the strategy, in presence and absence of a damping force, this experiment were implemented in MATLAB. The damping force was added into first equation differential of the model (3), as $-0{,}5\dot{\gamma}$. We have considered the physical parameters $\mu = 0{,}55$ and $\eta = 1{,}33$, and the controller parameters as $\beta = 1{,}002$ and $\alpha = 0{,}4252$. As far as the initial conditions are concerned we take $(\gamma, \dot{\gamma}, \theta, \dot{\theta}) = (0{,}8[rad], -0{,}05[rad/s], 0{,}3[rad], 0{,}1[rad/s])$.

Figure 2 and Figure 3 show the results coming out from the numerical simulations, the continuous lines correspond to the system response when the damping forces perturbation is absent, while the dotted line represent the case when this force is present. As can be seen, the state γ converges to zero faster that θ. This means that, while the angular position of the wheels is increased, the angle position of the pendulum approaches to zero. In other words, first, the controller brings the pendulum into small vicinity of zero, while the wheels angular position reaches its maximum, and second, the controller force to move the wheels slowly to the origin . Additionally, we can see, the effect of the damping force, where the closed loop response is still quite well. Observe that in order compensate the damping force effect, the wheels have to make larger displacement. Finally, Figure 4 shows the behavior of the control τ.

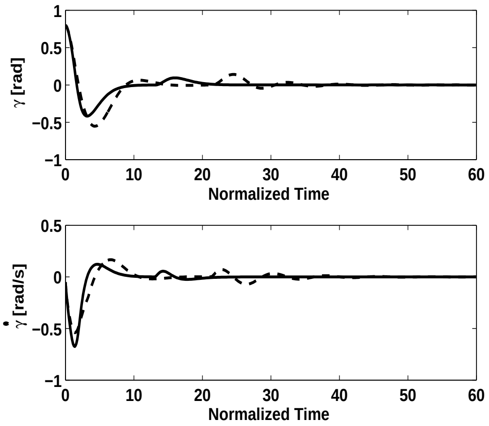

Figure 2. Closed loop response of the γ and $\dot{\gamma}$

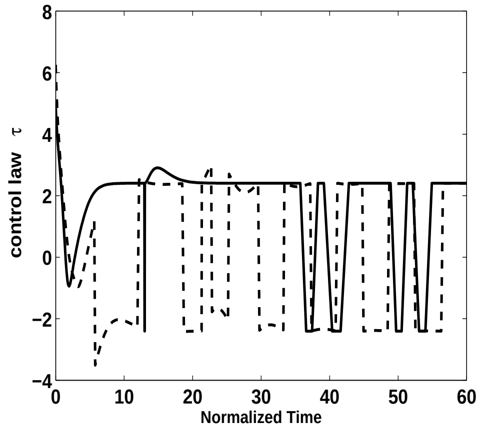

Figure 4. Depicts the behavior of the controller τ

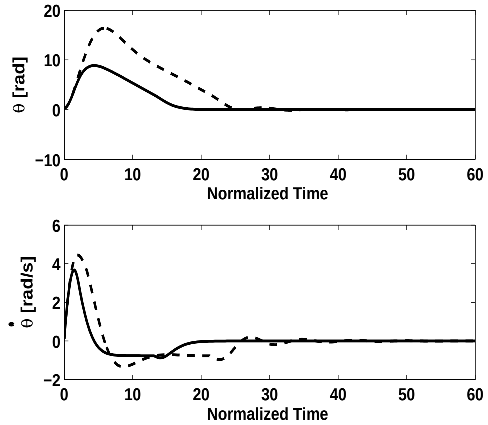

Figure 3. Closed loop response of the θ and $\dot{\theta}$

V. Conclusions

A nested saturation based controller allows us to solve a number of interesting non linear control stabilization problems. In this case, we have applied this technique for the stabilization of the **TWIP** under assumption that the pendulum is initialized in the upper half plane. The control strategy used a model that can be expressed approximately, as a nonlinearly perturbed chain of four integrators. Intuitively, the proposed controller consist of two stages. Firstly, we bring the pendulum close enough to the vertical unstable equilibrium point and then gradually the wheels position is moved to the origin. Also, this controller makes the system asymptotically stable and after some time finite, assures that all states converge exponentially to zero. Our the stability analysis is fairly simple because it is carried out using the Lyapunov method. Finally, the closed loop performance was tested by numerical simulations.

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